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ANALYSIS OF THE INFLUENCE  
OF INTERNAL WAVES OF TIDAL PERIOD  
UPON THE MIXED-LAYER DEPTH

HAROLD R. LAMBRIGHT

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OF TIDAL PERIOD UPON THE MIXED-LAYER DEPTH  
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Harold R. Lambright





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OF TIDAL PERIOD UPON THE MIXED-LAYER DEPTHS

by

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Lieutenant, United States Navy

Submitted in partial fulfillment of  
the requirements for the degree of

MASTER OF SCIENCE

United States Naval Postgraduate School  
Monterey, California

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## ABSTRACT

The apparently random oscillation of the mixed-layer depth is investigated and, by the nature of its energy spectrum and its correlation with tidal activity, a determination of the source of some components of this oscillation is made.

Actual and smoothed values of the mixed-layer depth at ocean station "P" in the Gulf of Alaska are subjected to spectral analysis for the presence of suspected tidal periodicities. To locate the source of the tidal influences, correlations between tidal activity on the shelf and a measure of the magnitude of the oscillation of the mixed layer at "P" are found for various lags. The amplitudes of the oscillations observed at the ocean station are compared to those calculated from a model proposed by Rattray.

The spectrum analysis shows large energy peaks at tidal periodicities. The close agreement, especially in the winter season, between the lag for large correlation of the tidal activity at the shelf with the mixed-layer depth oscillation at "P" and calculated travel times of the internal tide from the shelf to "P" suggest that the generation of the internal tide is at the continental shelf.



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## 1. Introduction

The occurrence of periodic oscillations of the thermocline, a region of large temperature gradient between the mixed, shallow layer and the more dense, deep layer of the ocean, has been known for many years. Recently, investigations have been undertaken to determine whether internal waves of tidal period were present in these oscillations. Not only their existence but also the mechanism of their generation have been the concern of many oceanographers.

Knauss [3] has observed marked periodic motion of neutrally buoyant floats within the depths of 500 and 2800 meters at 28-12N, 139-07W and 28-48N, 117-41W. However, it was impossible to distinguish any tidal component in these measurements.

Reid [9], in analyzing the bathythermograph data obtained from three vessels occupying stations 40, 160, and 320 nautical miles offshore on a  $240^{\circ}$  true bearing from Point Sur, California, has observed semi-diurnal tidal periods from the data of the nearshore station. Little evidence of tidal influences was present at the two outer stations, which suggested distortion and attenuation of the internal tide with distance from the shore.

Haurwitz [2], in re-examining previous theories on internal waves, considered the effect of the earth's rotation, previously neglected, in the formulation of an internal-wave-velocity equation. The earth's rotation increased the speed of the internal wave, for wave lengths similar to that of the tidal wave, to where it approached the speed of the surface tidal wave. Thus, inclusion of rotation gave a better explanation for resonance between the surface and internal tide.

Rattray [7] theorized that since large frictional damping was present,



resonance would not occur and that the generation of the internal tide depended on a coupling, at the continental shelf, between the surface and internal tide. The ocean boundaries have been suggested as the source of generation of internal waves of tidal period because of their large amplitudes near shore. Ratnayake [8] derived an equation showing the internal-wave amplitude to be dependent on the amplitude of the surface tide, the depth of the mixed layer, and the dimensions of the continental shelf. He [7] found a damping coefficient which varies directly with the square root of the eddy viscosity, and inversely with the square root of the density difference and the three-halves power of the depth of the shallow, mixed-layer.

The objective of this paper is threefold:

1. Investigate, for tidal periodicities, the oscillations of the thermocline observed at an offshore station, specifically, ocean weather station "P" (50N, 145W);
2. Test Ratnayake's theory that the generation of the internal tide is at the continental shelf;
3. Using Ratnayake's equation for generation, compute the amplitude of the internal wave at the shelf and test his theory for dissipation.





## 2. Determination of Tidal Periodicities in the Mixed-Layer Depth Oscillations.

Since the top of the thermocline is coincident with the depth of the upper, mixed-layer, the oscillations of interest will now be considered as those pertaining to the depth of the mixed-layer. No simple objective criterion was employed for the determination of the depth to which mixing occurred. Rather, the base of the isothermal (mixed) layer was assumed to be an interface whose depth varied rapidly (as a substantial surface) in response to the divergence associated mainly with internal waves and slowly as a consequence of vertical mixing. A subjective estimate of the depth of this interface was recorded as the depth of the mixed-layer.

The source of the data for the investigations was the bathythermograph record from ocean station "P". Positioned in the Gulf of Alaska at nearly a focal point to the stations along the coast (fig. 1.), ship "P" offered the optimum location for the observance of tidally induced activity at a distance from shore. If the internal tide were generated at the shelf, the energy, though subject to attenuation and angular spreading, would be concentrated (focused) at the ocean station.

Commencing August 06 and continuing through to September 01, 1961, 648 consecutive hourly records were examined and the values of the mixed-layer depths extracted. A Tukey spectrum analysis [1], [5] was performed on these depths utilizing program Blacky [4] for the 1604 Computer. A least-squares linear trend was removed, and the autocorrelations and smoothed spectral estimates computed. Fig. 2, a plot of the spectral estimates versus a variable,  $h$ , proportional to frequency<sup>1</sup>, shows that maximum energy is

---

<sup>1</sup>  $h = 2m \Delta t f$ ; where  $m$  is the maximum number of intervals per lag, and also the number of spectral bands determined in the analysis,  $\Delta t$  is the time interval between successive sampling points, and  $f$  is the frequency.





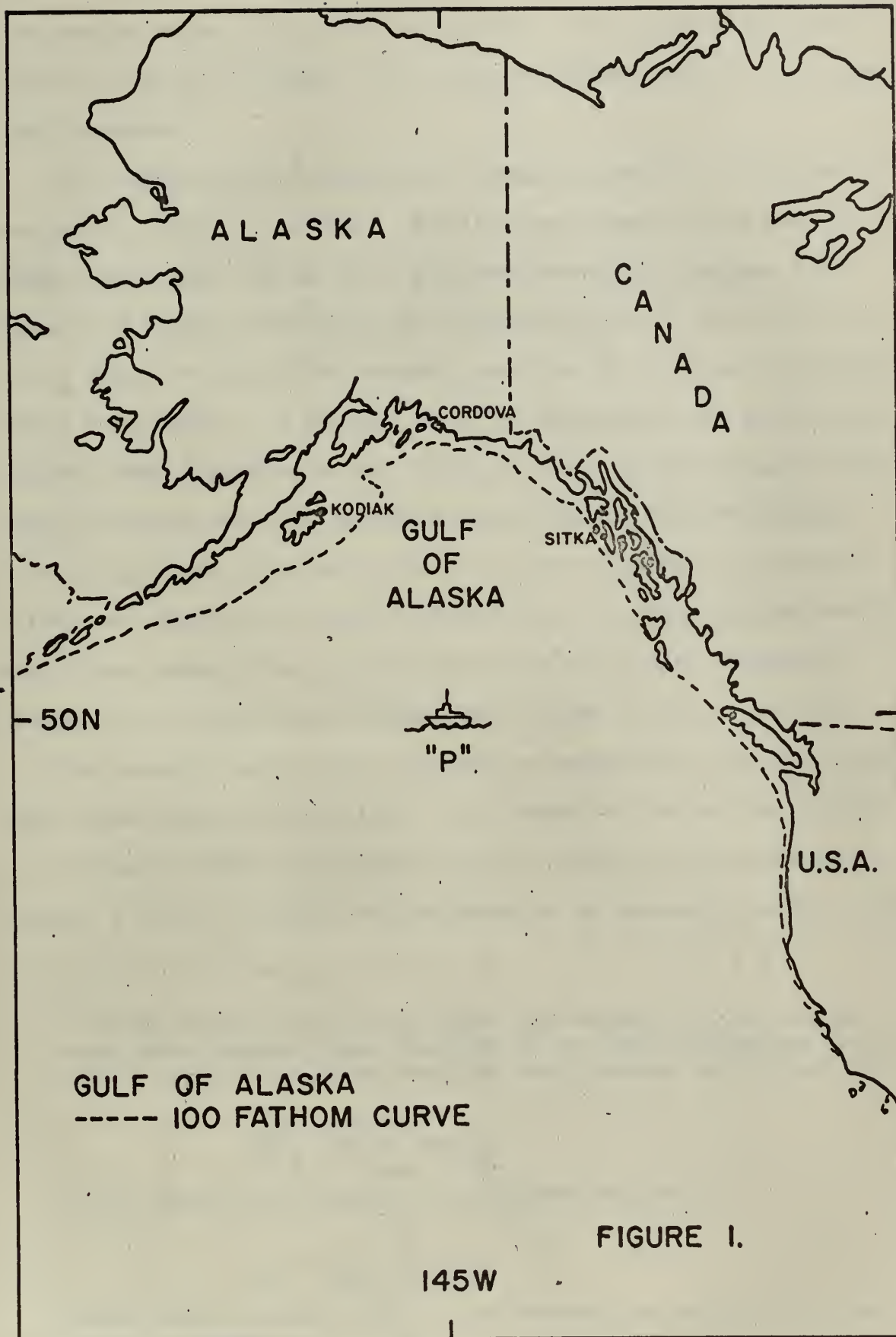


FIGURE I.



concentrated around the 12-hour period, close to the semi-diurnal tidal period. There is no evidence of an energy concentration near the diurnal tidal component.

Two different methods were used to determine whether a 14-day period was present in the oscillations. The first was applied to the actual mixed-layer depths from the daily 0200 bathythermograph readings for a duration of 90 days from July 01 to September 28, 1958. The second used the  $S_m$  values, which are the standard deviations of the actual mixed-layer depths with respect to a smoothed value<sup>2</sup> of the depth of the mixed layer. The data under investigation were the  $S_m$  values from July through December 1956 (160 values) and from September through October 1960 (60 values). By using  $S_m$  values, short-period variations were filtered out of the oscillations. Both types of data were subjected to a Tukey spectral analysis with a least-square linear trend removed; and the results, showing the presence of a 14-day maximum energy, were plotted in figs. 3, 4, and 5.

The secondary maximum which occurred at seven days in the mixed-layer depth values (fig. 3) was believed to be related to the cyclical changes in the pressure field at the ocean station. Pursuing this possibility further, a spectral analysis was performed on the mean daily surface pressure

---

<sup>2</sup> To smooth the values of the mixed layer depth, five-day running means were computed, using the 0200 and the 1500 observations on the day of interest and on the two days which preceded and followed it.

$$\overline{MLD}_m = \frac{1}{9} \sum_{j=-4}^{j=4} MLD_{m+j}$$

The  $S_m$  values were computed, over five-day intervals.

$$S_m = \left[ \frac{1}{10} \sum_{j=-4}^{j=4} Y_{m+j}^2 \right]^{1/2}$$

where  $m$  is the serial number of the observations and  $\frac{m}{2}$  that of the day of interest, and

$$Y_m = MLD_m - \overline{MLD}_m$$



at the ocean station during the corresponding period. The presence of relatively large energy in seven-day and eight-day periods in the pressure spectrum (fig. 3) and a coherence of 0.866 between the mixed-layer depth and surface pressure, for a seven-day period, suggested that such a relation does exist. Either the pressure system, directly, or its associated wind field, causes a divergent (convergent) flow in the surface waters, reducing (increasing) the depth of the mixed layer.

The Tukey spectrum analysis gives only estimates of the energy density, since the data records are of finite length. The values of these spectral estimates have been shown to be chi-squared distributed. Confidence limits, based on this distribution, were calculated for each of the spectra and inserted on the respective figures.

Program Blacky also computes the coherence squared between two variables. Analogous to a correlation coefficient between two random variables, the coherence squared is equal to the square of the correlation coefficient between two spectral variables. The coherence squared was computed (fig. 6) for hourly values of the mixed-layer depth from August 19 through September 01, 1961 (336 observations) against daily lags of the hourly tidal heights at Kodiak, Alaska. Though its value is small, there is a definite period variation in the coherence. A maximum for a 13-day lag may indicate the delay between the time that the energy was generated by the tides and its appearance at ship "P" as the energy in the mixed-layer oscillations. The scattering of the internal tide by currents and other internal waves, the interference from waves generated at other sources, and the attenuation of the internal tide along its travel path are processes which lead to the deterioration of the coherence between these two spectral variables.





ENERGY-DENSITY SPECTRUM (August 06 Through September 01, 1961)

$h = 2m \Delta t f$

$N = 648 \quad \Delta t = 1 \text{ Hour} \quad m = 30$

90% Confidence Limits

$0.73 [A(h)]^2 \leq [A(h)]^2 \leq 1.48 [A(h)]^2$

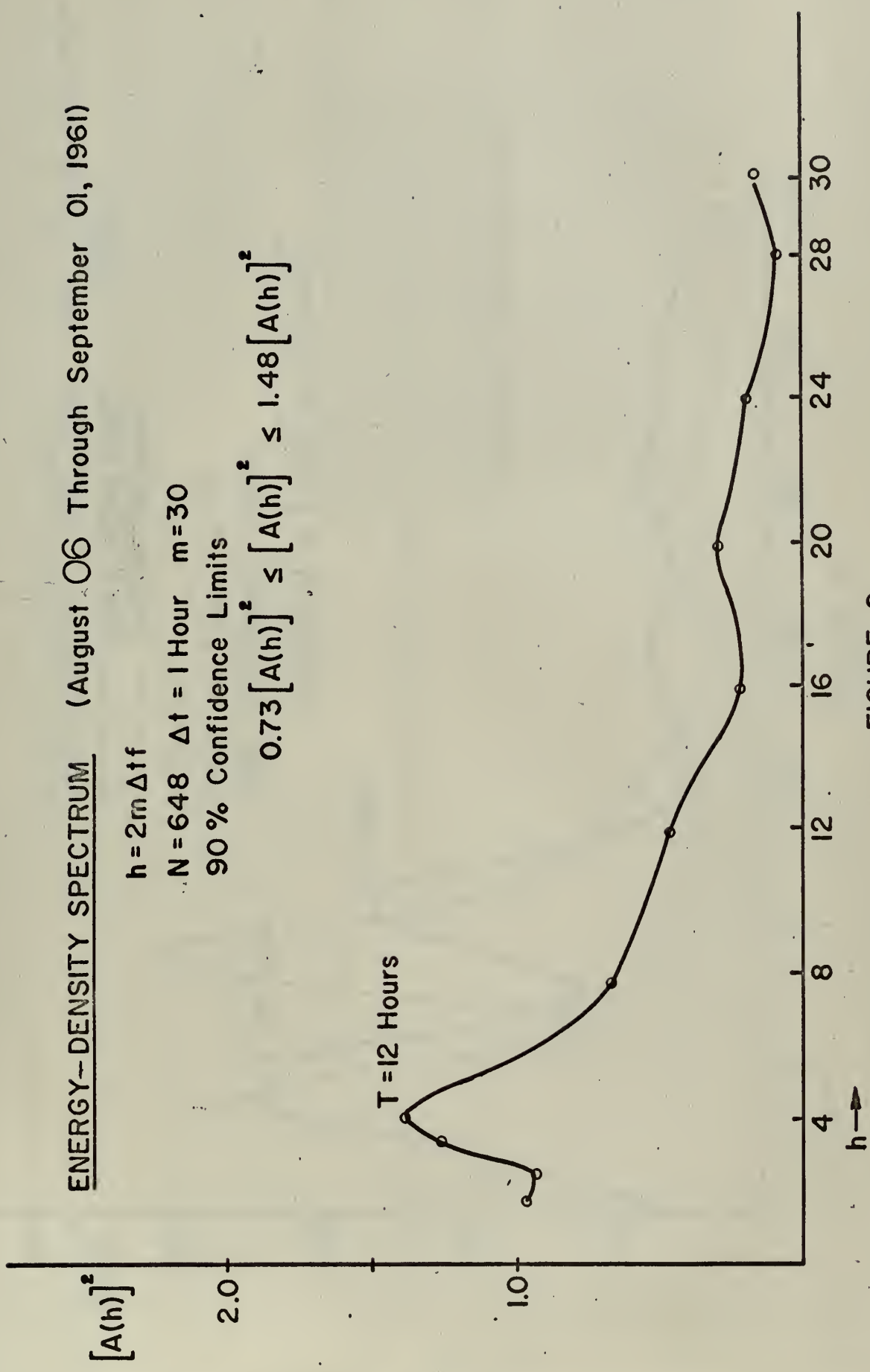


FIGURE 2





# ENERGY-DENSITY SPECTRUM (July Through September 1958)

—○— Mixed Layer Depth  
 -\*- Surface Pressure

$h=2m \Delta t f$

$N=90 \Delta t=1 \text{ Day } m=28$

95% Confidence Limits

$$.42[A(h)]^2 \leq [A(h)]^2 \leq 4.8[A(h)]^2$$

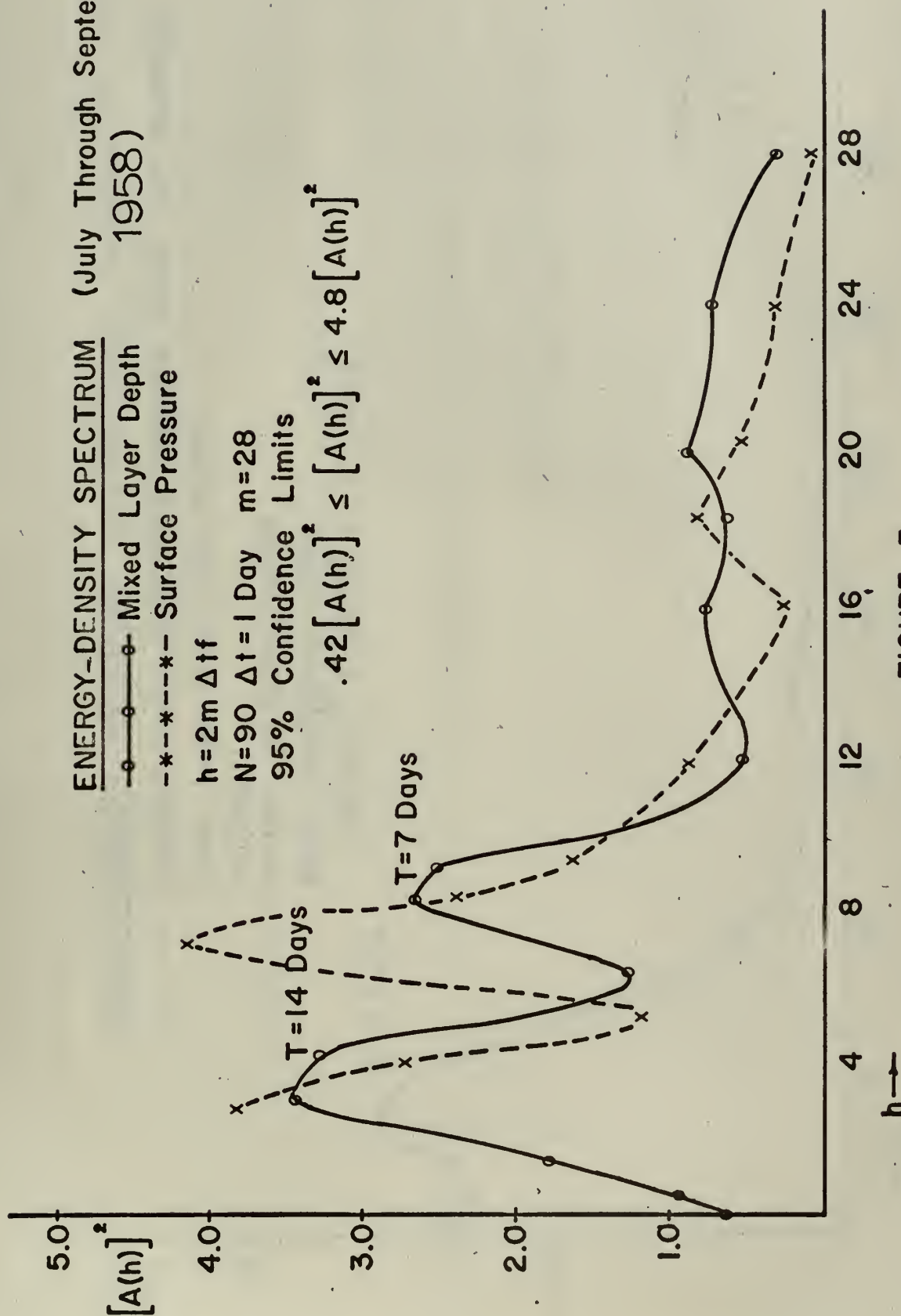


FIGURE 3



# ENERGY-DENSITY SPECTRUM (July 03 Through December 09, 1956)

—○— Standard Deviations Of The Observed Minus The Smoothed  
Mixed Layer Depths ( $S_m$  Values).

$h = 2m \Delta t f$

$N = 160 \quad \Delta t = 1 \text{ Day} \quad m = 28$

90% Confidence Limits

$$.56[A(h)]^2 \leq [A(h)]^2 \leq 2.45[A(h)]^2$$

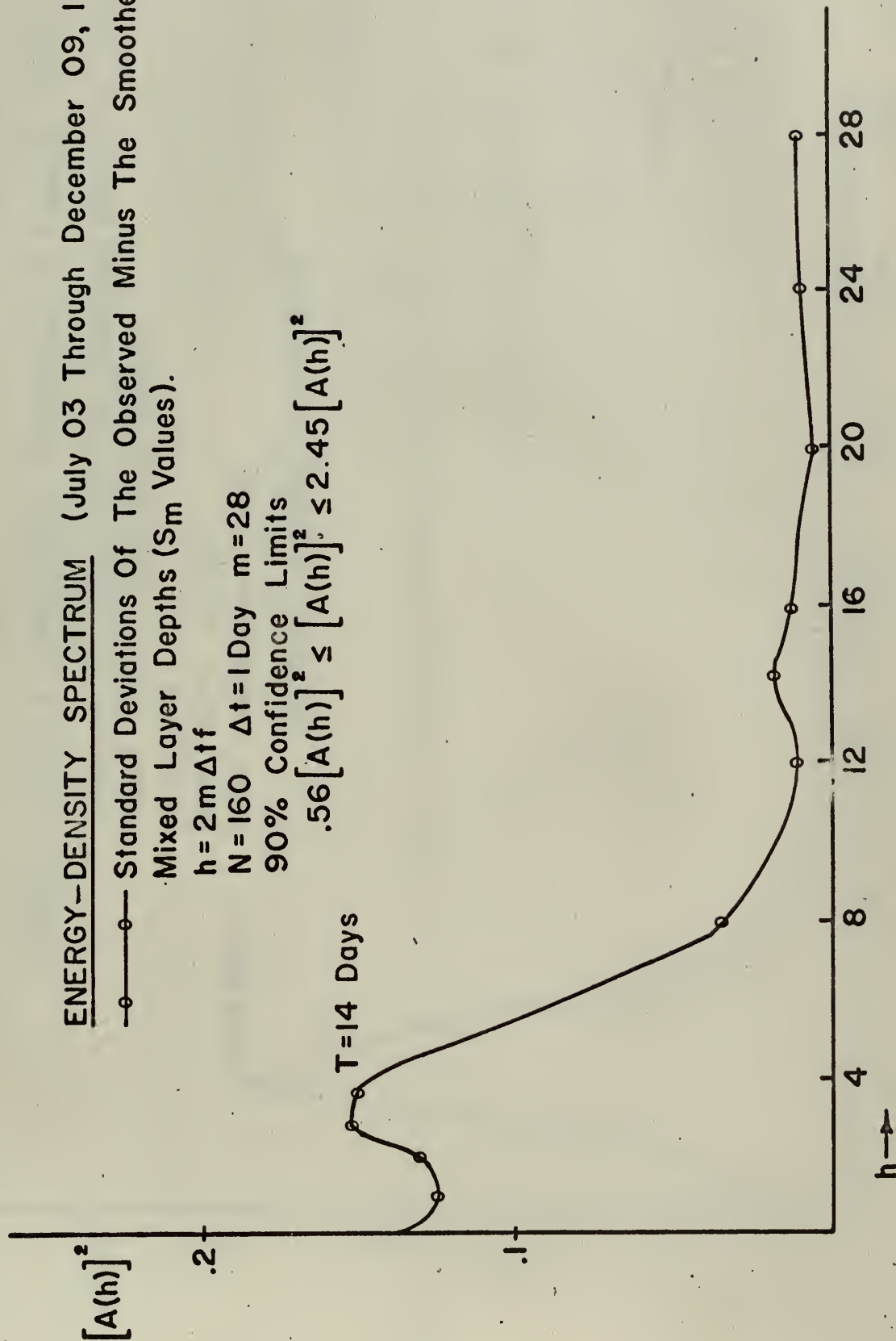


FIGURE 4



# ENERGY-DENSITY SPECTRUM (September 01 Through October 30, 1960)

—○— Standard Deviations Of The Observed Minus The Smoothed  
Mixed Layer Depths ( $S_m$  Values).

$h = 2m \Delta t f$

$N = 60 \quad \Delta t = 1 \quad m = 28$

95% Confidence Limits

$$.36[A(h)]^2 \leq [A(h)]^2 \leq 8.3[A(h)]^2$$

$T = 14$  Days

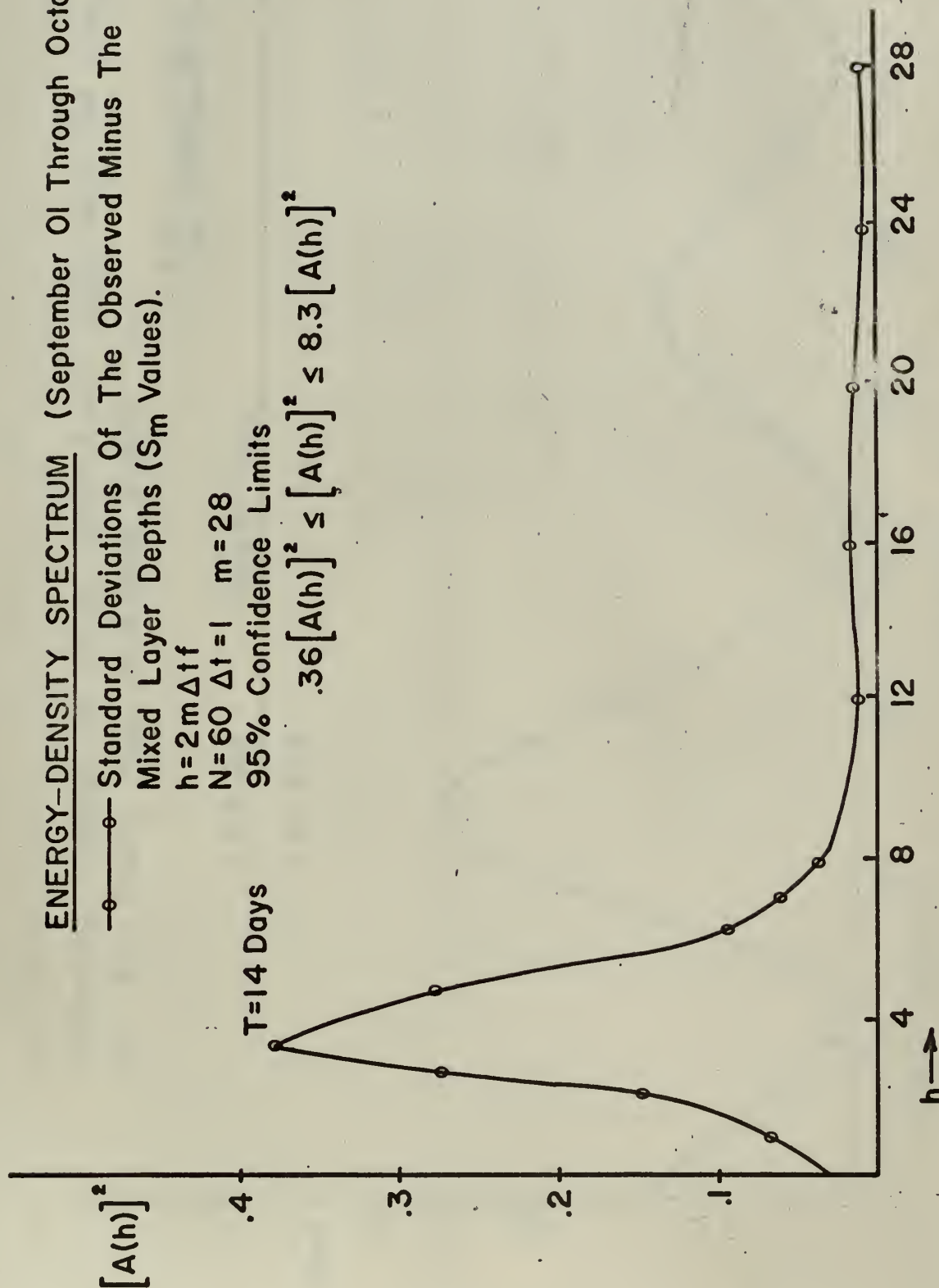


FIGURE 5



# Coherence Squared Of Hourly Mixed-Layer Depth And The Lag Of The Hourly Tide At Kodiak, Alaska

04 Jul N=336  
Hourly Tide  
19 Aug Hourly MLD  
01 Sep 1961  
01 Sep 1961

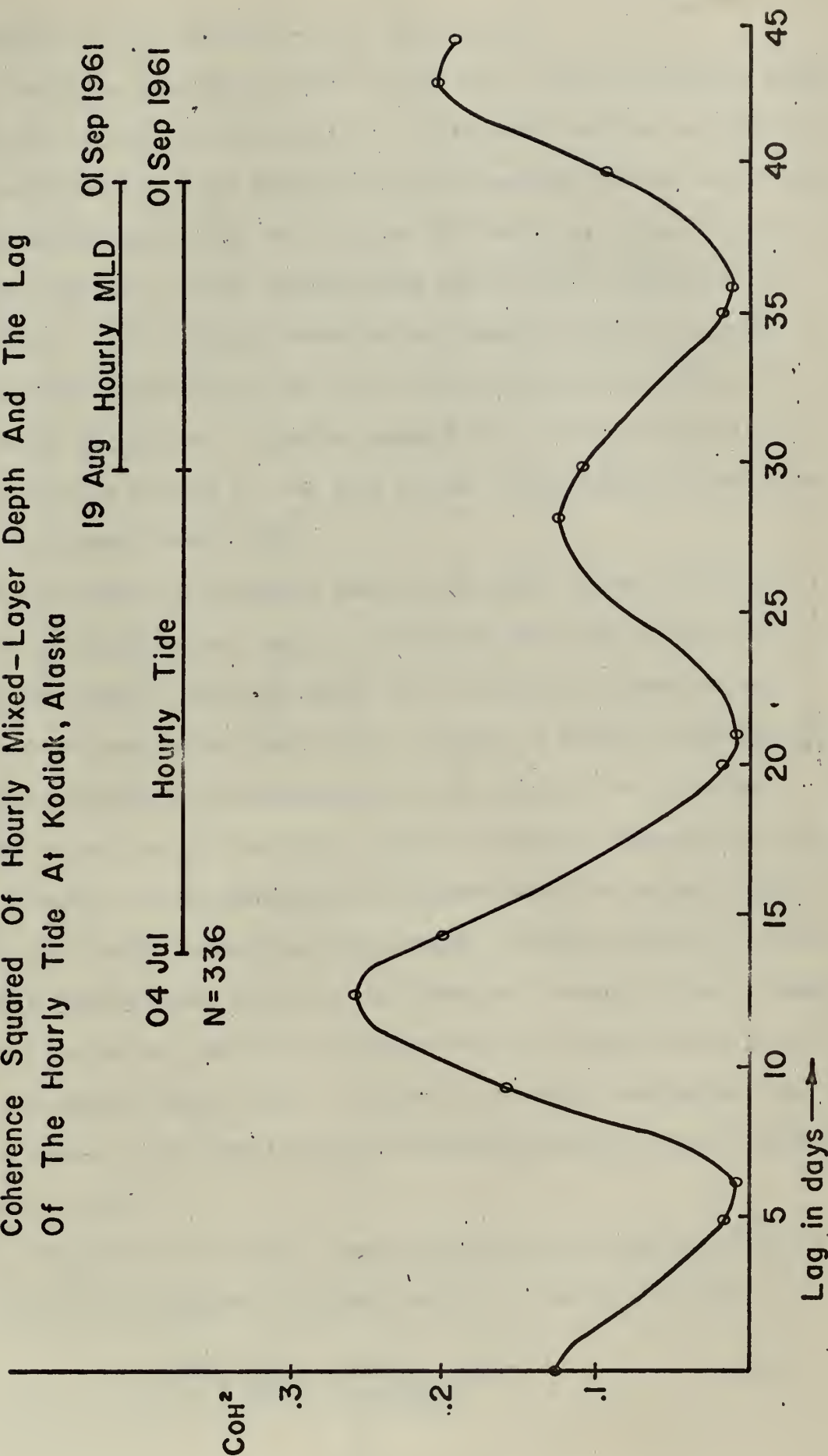


FIGURE 6







### 3. Generation of the Internal Wave of Tidal Period

To locate the generation of the internal tide at the continental shelf, correlations between the tidal activity on the shelf and the oscillations of the mixed layer at ocean station "P" were found for various lags. The parameter representing tidal activity was the daily tide range observed at coastal stations, and the internal-wave activity was represented by the  $S_m$  values. Then the phase speed of the internal wave was computed for a two-layer system (equ.1) and its travel time from the shelf to the ocean station determined. A scatter diagram (fig. 9) was constructed to show the relation between the lag time of the initial maximum correlation and the calculated travel times.

The tidal ranges at Cordova, Kodiak, and Sitka, Alaska are all in phase and the stations are roughly equidistant from "P", so that the internal-wave energy from each should reach the ship at about the same time. But the continental shelf in the vicinity of Kodiak Island appears to be more favorable to the generation of the internal tide (further discussed in section 4). Therefore, the tidal range at Kodiak alone was used as a measure of the generation of internal waves on the shelf edge.

The correlation between the tidal ranges at Kodiak and the  $S_m$  values for "30-day" intervals was found for lags from zero through 45 days. Summer and winter months were selected to illustrate the different characteristics of their respective mixed layers. Figures 7 and 8 are correlograms typical of these seasons. The results of the correlations from all the investigations are shown in table 1.

The evaluation of the phase speed was based on an equation given by Proudman [6], for progressive internal waves in a two-layer system:

$$C = \left[ \frac{\rho - \rho'}{\rho} g \frac{h'h}{h' + h} \left( \frac{1}{1 - \left( \frac{T}{T_p} \right)^2} \right) \right]^{1/2} \quad (1)$$



where  $g$  is the acceleration of gravity;  $\rho'$  and  $\rho$  the densities of the upper and lower layers, respectively;  $h'$  and  $h$  their depths;  $T$  the period of the wave; and  $T_p$ , the period of one-half a pendulum day, equal to  $12/\sin \phi$  hours, where  $\phi$  is the latitude.

Assuming the upper, mixed-layer ( $h'$ ) to be much shallower than the lower layer ( $h$ ) and substituting values for a mean latitude of  $54^\circ\text{N}$ , the acceleration of gravity equal to 9.8 meters per second per second, and  $h'$  in meters<sup>3</sup>, the phase speed equation, in knots became

$$C = 11.2 \left[ \left( \frac{\rho - \rho'}{\rho} \right) h' \right]^{1/2} . \quad (2)$$

Salinity values were obtained from the Manuscript Report Series of the Fisheries Research Board of Canada and density values were calculated from the tables by Knudsen. The phase speeds and the travel times over the 465 nautical mile path from the shelf to the ocean station were computed for each of the 30 day intervals and the results entered in table 1.

By the construction of a scatter diagram (fig. 9), a good agreement is seen to exist between the lag of the initial maximum correlation and the computed travel times during the winter months. This diagram is discussed at length in section 5.

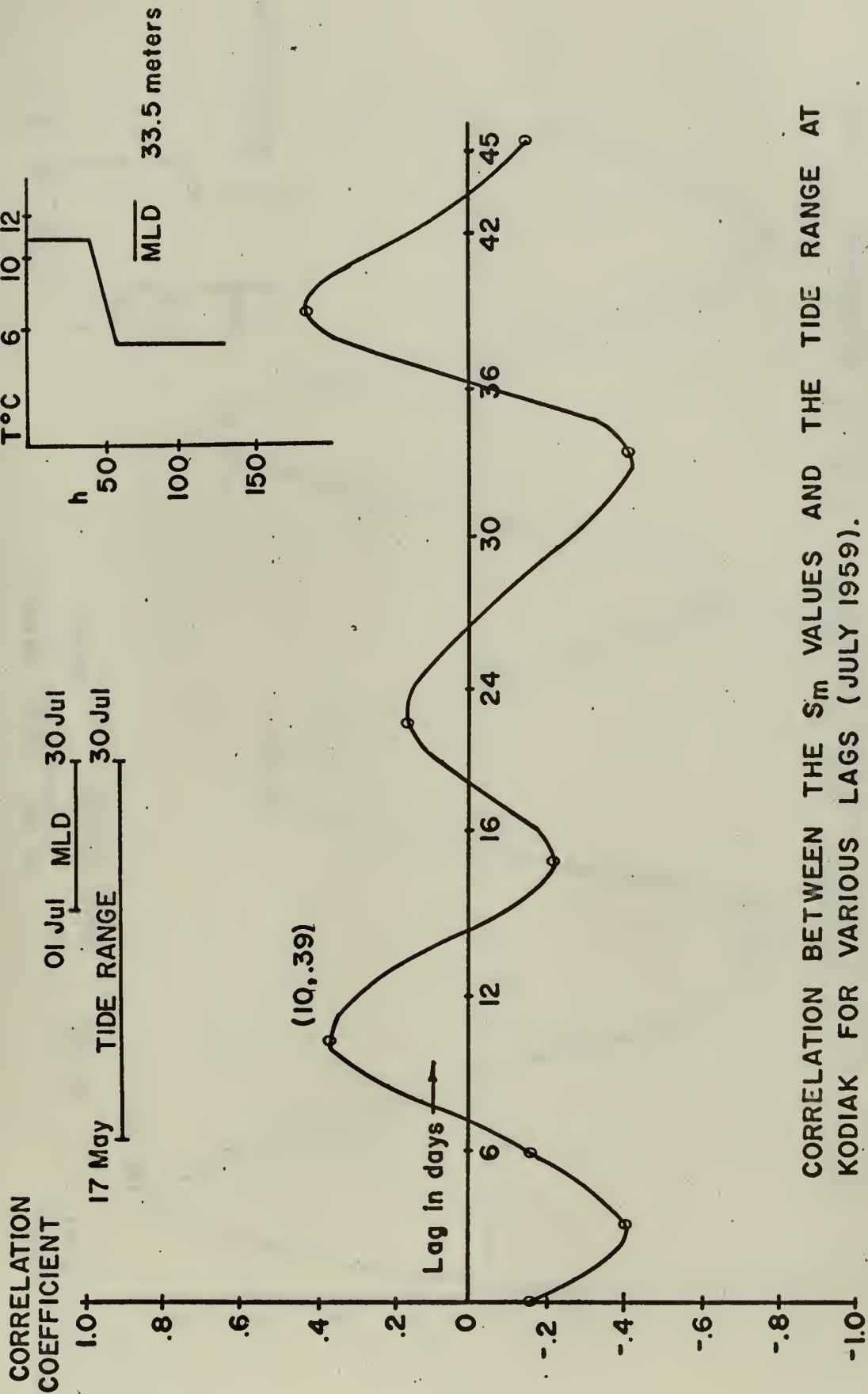
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<sup>3</sup>  $h'$  is measured at "P" and is averaged for 30-day intervals. In reality,  $h'$  varies along a path from the shelf to "P" and the travel time should be derived from

$$\int_0^R \frac{dx}{c[h'(x)]} = \int_0^R \frac{dx}{c(x)} ,$$

where  $R$  is the travel distance.



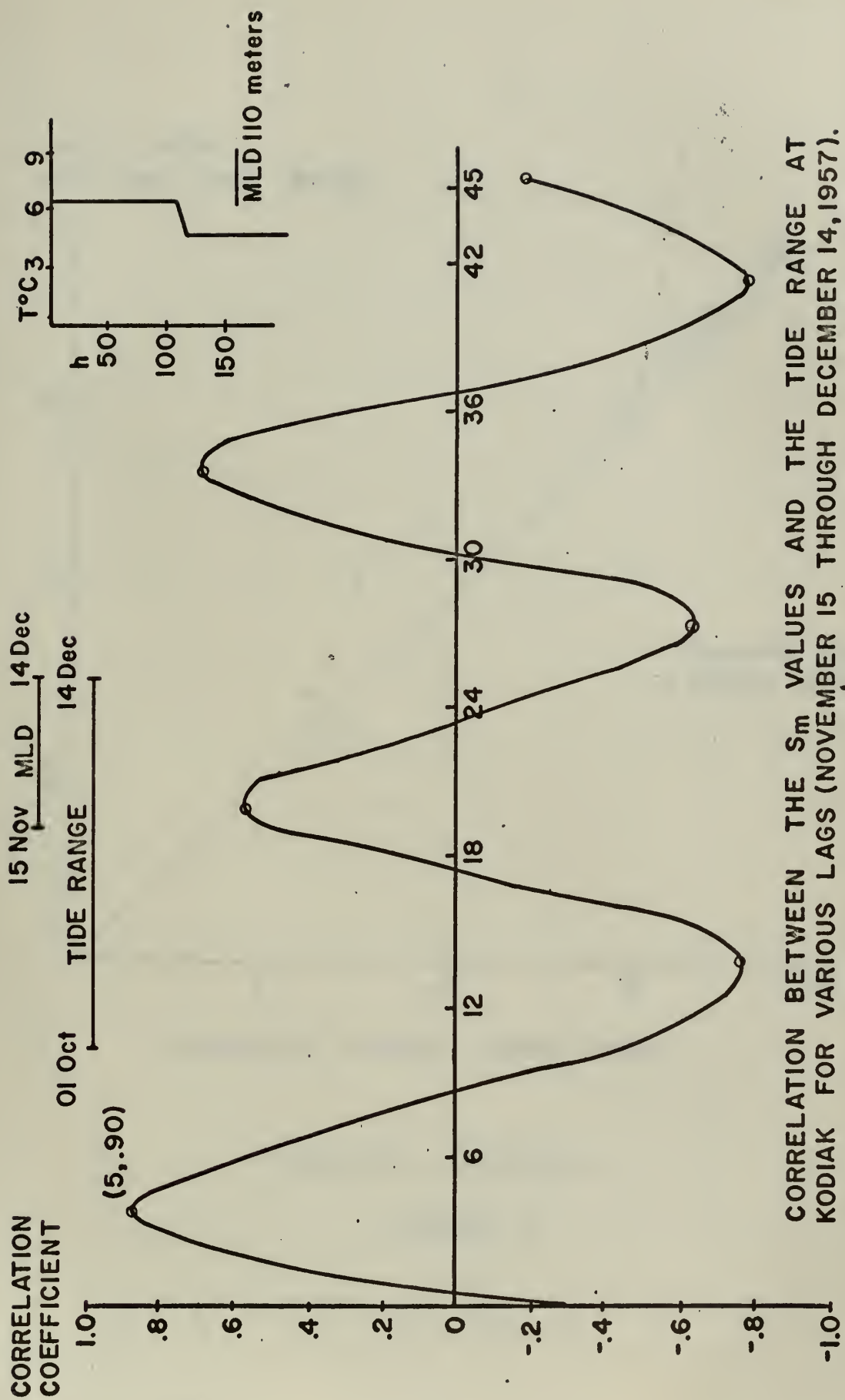


CORRELATION BETWEEN THE  $S_m$  VALUES AND THE TIDE RANGE AT KODIAK FOR VARIOUS LAGS (JULY 1959).

FIGURE 7





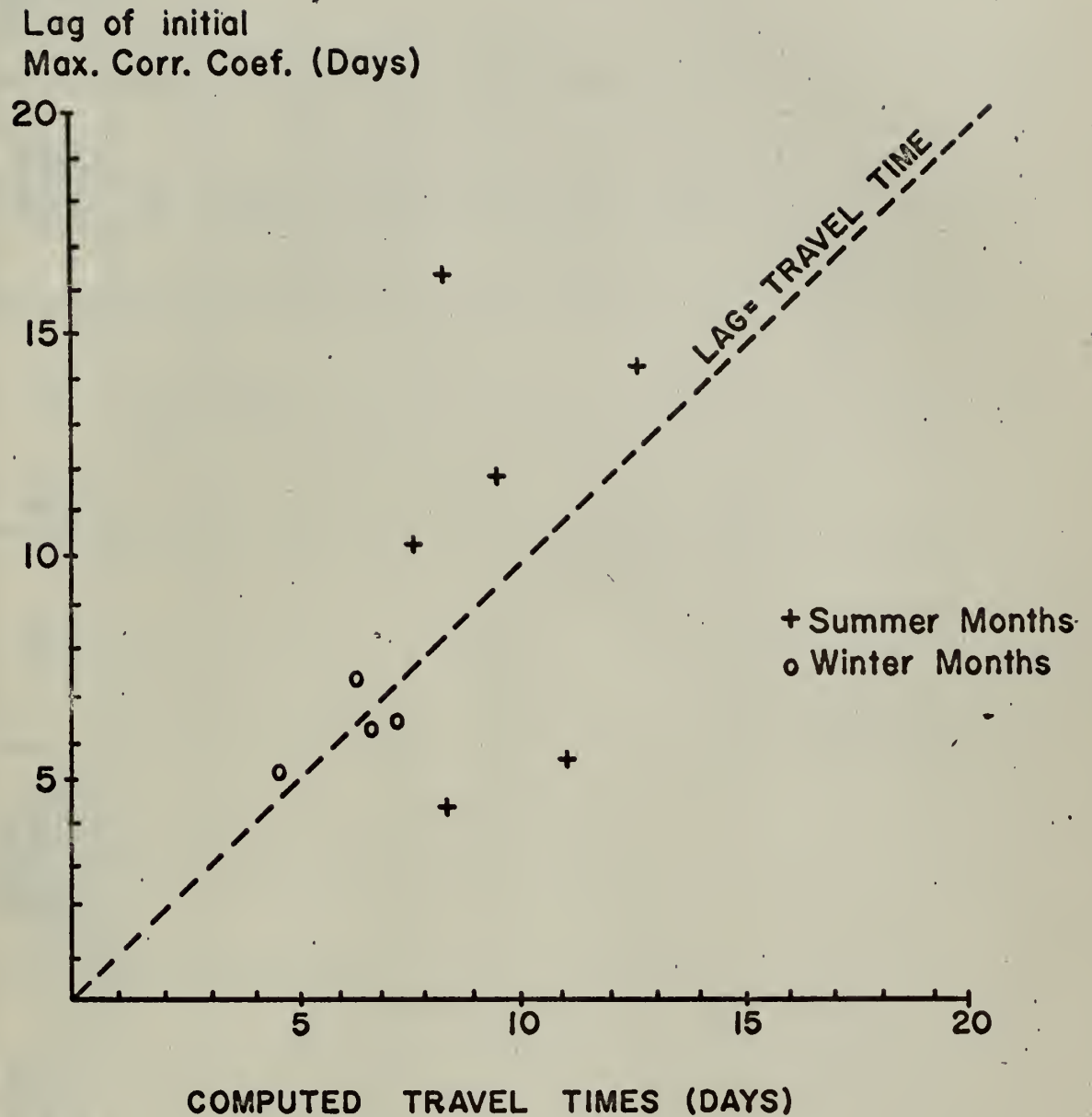


CORRELATION BETWEEN THE  $S_m$  VALUES AND THE TIDE RANGE AT KODIAK FOR VARIOUS LAGS (NOVEMBER 15 THROUGH DECEMBER 14, 1957).

FIGURE 8







SCATTER DIAGRAM

FIGURE 9

MARSAID JETTIES

OS                      ei                      oi                      e

Legend:  
 + Summer Months  
 o Winter Months

Month Type	Days of Travel (X)	Days of Stay (Y)
Summer	10	5
Summer	20	15
Summer	30	25
Summer	40	35
Summer	45	40
Winter	15	10
Winter	25	20
Winter	35	30
Winter	40	35
Winter	45	40

Max. Corr. Cost. (Days)      Leg of initial

INTERVAL	MEAN MLD (h') (METERS)	INITIAL MAXIMUM CORRELATION COEFFICIENT	LAG (DAYS)	$\frac{P P'}{P} \times 10^{-3}$	PHASE SPEED (KNOTS)	COMPUTED TRAVEL TIMES (DAYS)
Jul 03 - Aug 01 1956	22.1	.62	12	1.63	2.13	9.1
Nov 15 - Dec 14 1956	82.0	.36	6	.975	3.18	6.1
Aug 01 - Aug 30 1957	22.1	.77	16	1.86	2.27	8.5
Nov 15 - Dec 14 1957	110.0	.90	5	1.29	4.20	4.6
Jun 01 - Jun 30 1958	16.6	.32	14	1.23	1.58	12.4
Nov 09 - Dec 08 1958	84.0	.39	6	1.14	3.10	6.25
May 01 - May 30 1959	31.3	.47	5	.875	1.88	10.25
Jul 01 - Jul 30 1959	33.5	.39	10	1.65	2.65	7.3
Jun 03 - Jul 02 1960	40	.5	4	1.26	2.50	7.75
Dec 01 - Dec 30 1960	97.2	.2	7	.85	3.20	6.0

TABLE 1



#### 4. Comparison of Observed Internal Waves with Those Predicted by Ratray's Model.

Ratray [7] derived an equation for the amplitude of the internal tide at its point of generation, the continental shelf. His theory applied to a two-layer system in which no variations were present in the properties of the ocean and the bottom topography in a direction parallel to the coastline. A model, where the shelf depth increases linearly with distance from the shore, was considered; and the amplitude, B, was determined by the expression:

$$B = \zeta_o' h' \left( \frac{1}{d} - \frac{1}{D} \right) \left[ \frac{K_2 J_0(2K_1 l) - \frac{K_2}{K_1} J_1(2K_1 l)}{[J_0(2K_1 l)]^2 + \frac{K_2}{K_1} [J_1(2K_1 l)]^2} \right]^{1/2} \cdot e^{i(K_2 l - \mu + \frac{\pi}{2})} \quad (3)$$

where  $\zeta_o'$  is the surface tide amplitude;  $h'$  the depth of the upper mixed layer;  $d$  the depth of the shelf;  $D$  the depth of the ocean,  $l$  the width of the shelf and

$$K_1^2 = \frac{(\sigma^2 - 4\omega^2) \frac{d}{h'(d-h')}}{g \frac{\Delta\rho}{\rho}} \quad (4)$$

and

$$K_2^2 = \frac{(\sigma^2 - 4\omega^2) \frac{D}{h'(D-h')}}{g \frac{\Delta\rho}{\rho}} \quad (5)$$

where  $\sigma$  is the angular frequency of the wave and  $\omega$  the angular velocity of rotation of the ocean area.  $\mu$ , the variation with time, is determined by:

$$\mu = \tan^{-1} \left[ \frac{K_2}{K_1} \frac{J_1(2K_1 l)}{J_0(2K_1 l)} \right] \cdot$$





The ratio of the amplitude of the internal tide at a distance from the shelf to its original amplitude at the shelf was determined by the expression:

$$\frac{B}{B_0} = e^{-\gamma x} \quad (6)$$

where  $\gamma$  is a damping coefficient; and  $x$ , the distance traveled.

Rattray [7] has investigated theoretically the frictional effects on the propagation of the internal tide, finding an expression for the damping coefficient,  $\gamma$ , for a two-layer system:

$$\gamma = \left( \frac{\nu}{2g \frac{\Delta \rho}{\rho}} \right)^{1/2} \left( \frac{h' + h''}{h' h''} \right)^{3/2} \left[ \frac{(\sigma - 2\omega)^{3/2} + (\sigma + 2\omega)^{3/2}}{4\sigma} \right] \quad (7)$$

where  $\nu$  is the eddy viscosity; and  $h''$  the depth of the deep layer.

The shelf dimensions at Cordova, Kodiak, and Sitka, Alaska were extracted from the Contoured Position-Plotting Sheets published by the U. S. Hydrographic Office. Assuming a mixed-layer depth of 100 meters and a surface-tide amplitude of 1.5 meters, the evaluation of equation 3 yielded 1.05, 2.45, and .69 meter amplitudes at the respective stations. For a mixed-layer depth of 30 meters, the amplitude of the internal wave at the shelf near Kodiak was 1.9 meters.

Since the energy in a wave is proportional to the square of the wave amplitude, the internal tide at Kodiak has nearly six times the energy than that generated at Cordova and 12 times that at Sitka. For this reason, tidal activity at Kodiak only was considered in the correlations with the  $S_m$  values (section 3).

Substituting a mixed-layer depth of 30 meters and a constant eddy viscosity of one centimeter squared per second, the computed damping





coefficient,  $\gamma$ , near Kodiak, was  $3.15 \times 10^{-3}$  per kilometer (eq. 7). Using this value for  $\gamma$  and 865 kilometers as the distance traveled to ocean station "P", the attenuated amplitude of the internal tide is 0.12 meters, assuming no losses by scattering and no divergence.

A comparison of the calculated amplitude of the wave affected by friction, 0.12 meters, to the amplitude determined by the spectral analysis, 3.0 meters, indicated that only a small portion of the energy in the tidal band at "P" was contributed by the internal tide generated on the shelf near Kodiak.

To find the energy in the semi-diurnal period at "P", at Tukey spectrum analysis was performed on the 648 hourly mixed-layer depths from August 06 through September 01, 1961, which had a mean layer depth of approximately 30 meters. As suggested by Munk [4], the integration for the 12-hour energy was over the frequency band  $\frac{1}{20} \leq f \leq \frac{1}{8.57}$  where  $f = \frac{h}{2m\Delta t}$ . The result, 4.5 meters squared, was the energy which would have been propagated by a wave with an amplitude of 3.0 meters.



## 5. Conclusions

The spectral analysis of the actual mixed-layer depth and the  $S_m$  values gave conclusive evidence of the tidal periodicities in the oscillations of the mixed layer at ocean station "P". In a few instances, the analysis showed high energies occurring at very low frequencies. Long period trends of a degree higher than one, but not associated with internal waves, seem to have contributed to these high energies.

The close agreement between the lag of the initial maximum correlation ( $S_m$  values versus tidal range) and the computed travel times for internal waves is consistent with Rattray's theory of the shelf generation of the internal tide. The correlograms of the hourly tides and the  $S_m$  values indicated poor correlation at zero lag. Therefore, it is concluded that the surface tide, which occurs at "P" at essentially the same time as at the shelf, has little, if any, direct effect on the variations of the mixed-layer depth; this result is in conflict with Haurwitz's theory of linkage between the surface and internal tides.

The scatter diagram (fig. 9) shows that the winter months gave the best agreement between lag and travel time; this is reasonable since the stable, deep, mixed layer better approximated the two-layer model used to estimate the travel time. The summer months had shallow mixed layers which were influenced by wind mixing, and both temporal and spacial variations in the depth of the mixed layer along a path from the shelf to the ocean station are known to exist. Though these could have an important influence on the travel time, the depth of the mixed layer was assumed constant from the shelf to the ocean station. An approach using a grid system with observed mixed-layer depths at each grid point along the travel path, would give more accurate results for the travel times.



From Rattray's theory on attenuation (eq. 7), the damping coefficient increases as the depth of the mixed-layer decreases, thus, the internal tide amplitude in the summer should have greater dissipation and have affected the oscillations of the mixed layer to a lesser extent. This situation, if true, would have exhibited itself in the correlation between the tide ranges and the  $S_m$  values. However, the correlation coefficients did not reflect the effects of attenuation, as their magnitude differed little from the correlations during the winter season.

Further examination of the attenuation model (eq. 7) revealed the importance of the selection of the eddy viscosity, since the damping coefficient is proportional to the square root of its value. With eddy viscosity one centimeter squared per second, the wave generated at the shelf was attenuated to six percent of its original amplitude, which contributes 0.16 percent to the observed 12-hour energy in the mixed-layer depth oscillations. It is difficult to determine if the portion of the energy which is associated with the calculated attenuated wave is of the correct order of magnitude. Only if the energy distribution is calculated continuously along the coast and a method is devised to discriminate the energy contributions converging from all directions on ship "P", can a reasonable statement be made about the magnitude of attenuation of a single wave generated at a designated location on the shelf.





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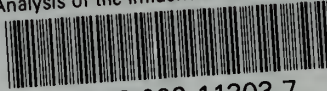






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